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**LETTER TO THE EDITOR**

# Multifractal diffusion in NASDAQ

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**Abstract**
It is shown that fluctuations of NASDAQ increments exhibit two scaling intervals: one (for relatively short-time increments, up to five months) is quasi-Brownian and another (for relatively long-time increments) is multifractal. For the multifractal diffusion a new type of scaling symmetry has been observed.

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In the last decade financial time series have been actively studied with methods which are used in physics (see, for instance, [1–12] and references therein). There have been scaling hypothesis, fractal and multifractal analyses of increments of the prices (or, in financial terms, returns). It is clear now that the returns of most indexes (such as S&P 500, DJ, CAC 40, Topix etc) can be multifractal [5–12]. It is curious that the multifractal analysis was applied to so-called 'old economy' indexes, while an index such as NASDAQ, which represents the new type of index, is still outside this consideration. The principal difference between the 'old' and 'new' economics is that the 'new economy' companies are supposed to compensate for their lack of present earnings by a great potential growth. The expectation of future earnings, that motivates the average investor rather than present reality (which, in contrast, is significant for the 'old economy'), results in index dynamics significantly different from the dynamics of the 'old economy' indexes (see [12] for an excellent comparative analysis performed from a physical point of view). Therefore, one can expect that the 'new economy' indexes (e.g. NASDAQ) reflect the differences in the underlying mechanisms. The main motivation of this paper is to investigate (a) in what time horizons these differences in the underlying mechanisms become crucial and (b) what new type of multifractal behaviour (if any) characterizes the 'new economy' indexes.

Figure 1 shows (in log–log representation) the dependence of the moments of NASDAQ increments (monthly returns for the period 1984–2000)

$$\Delta Z = |Z(t + \Delta t) - Z(t)| \tag{1}$$

on the time increment $\Delta t$. Upper sets of the data correspond to the moments of larger order $p$ ($p = 1, 2, \ldots, 7$). Straight lines (best fit) indicate the scaling

$$\langle \Delta Z^p \rangle \sim \Delta t^{\zeta_p}. \tag{2}$$

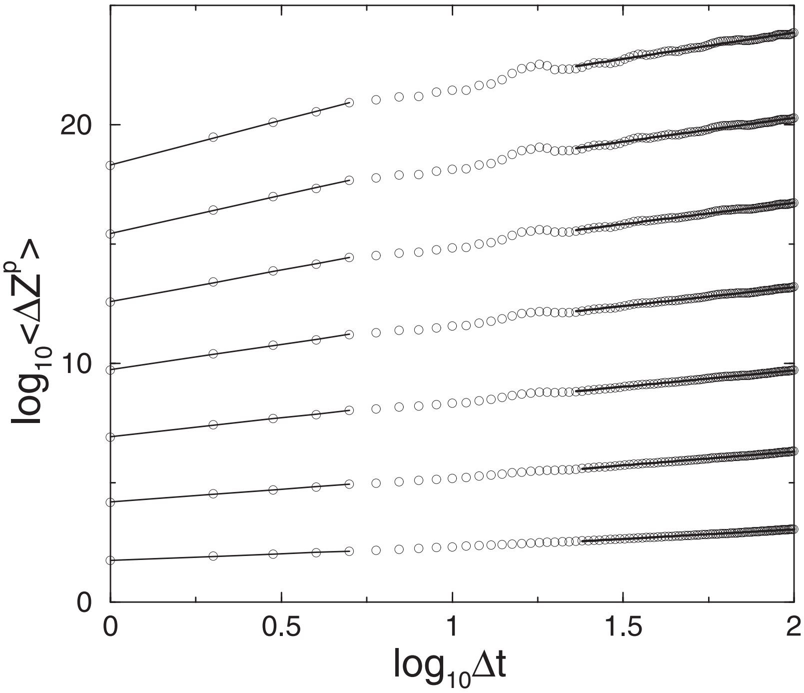


**Figure 1.** Dependence of the NASDAQ increments (monthly returns) $\Delta Z$ on the time increments $\Delta t$ in a log–log scale for moments of different orders $p = 1, 2, \ldots, 7$ (upper sets of the data correspond to larger values of $p$). The straight lines indicate scaling (2).

One can see two scaling intervals: one for comparatively short-time increments (up to five months) and another for long-time increments (more than 1.5 years). Corresponding values of so-called generalized Hurst exponents $H_p = \zeta_p/p$ are shown in figure 2: open circles correspond to short-time increments and filled circles correspond to long-time increments. Log–log scales are used in this figure to compare with power-law dependences (see below).

One can conclude from figure 2 that for the short-time increments (open circles) we have to deal with quasi-Brownian fluctuations with

$$H_p \simeq 0.54. \tag{3}$$

This observation is significant for financial applications because the quasi-Brownian behaviour of returns is a necessary condition for applicability of the Black–Scholse theory of the option prices (see [13] for a good physical introduction). Thus, one can conclude that this necessary condition is satisfied for the NASDAQ returns for the comparatively short-time periods. However, for the second scaling interval ($\Delta t > 1.5$ years) one can see in figure 2 (filled circles) strong deviations from the Brownian diffusion and, therefore, one can expect that for the long-time periods the Black–Scholse theory cannot be applied to NASDAQ. Actually, in this case we have to deal with multifractal diffusion (the exponent $\zeta_p$ is a nonlinear function of $p$). To develop an option pricing theory for this interval of timescales is an interesting problem for future investigations. Symmetries of the multifractal scaling could be useful for this purpose. Let us describe one such scaling symmetry. Let consider a generalized scaling

$$\langle \Delta Z^p \rangle \sim \langle \Delta Z \rangle^{\zeta^\star(p)} \tag{4}$$

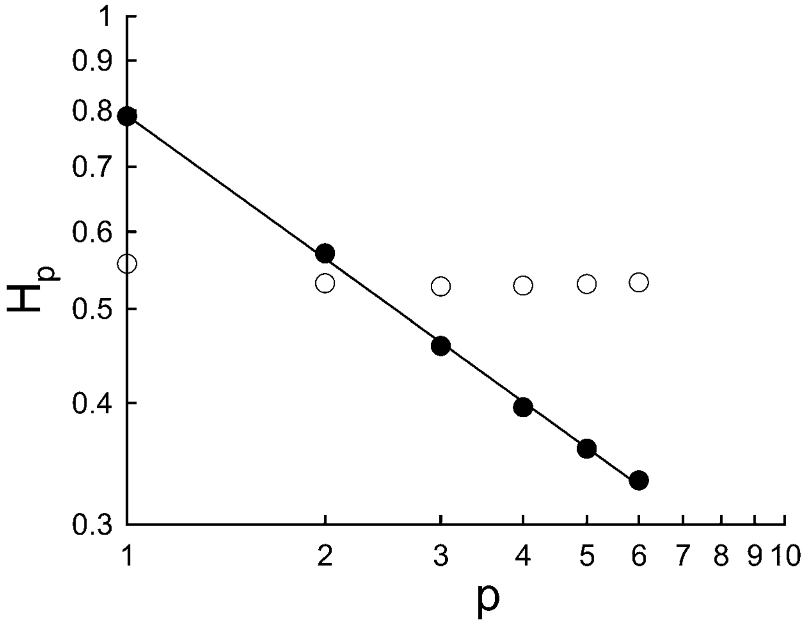


**Figure 2.** Dependence of log $H_p$ on log $p$. Open circles correspond to short-time scaling and filled circles correspond to long-time scaling. The solid straight line (best fit) is drawn to indicate the multifractal dependence (12) with $\gamma \simeq 0.556$.

where the exponent $\zeta^\star(p)$ is some function of the order $p$. Let us consider the map

$$\Delta Z \to \Delta Z^\lambda \tag{5}$$

(where $\lambda$ is an arbitrary real number) and define a scaling symmetry related to the map (5) as

$$\langle(\Delta Z^\lambda)^p\rangle \sim \langle(\Delta Z^\lambda)\rangle^{\zeta^\star(p)} \tag{6}$$

(cf (4)). From (4)–(6) we can find the generalized exponent $\zeta^\star(p)$ corresponding to the scaling symmetry related to the map (5). Indeed,

$$\langle\Delta Z^{\lambda p}\rangle \sim \langle\Delta Z\rangle^{\zeta^\star(\lambda p)} \tag{7}$$

and

$$\langle\Delta Z^{\lambda p}\rangle = \langle(\Delta Z^\lambda)^p\rangle \sim \langle\Delta Z^\lambda\rangle^{\zeta^\star(p)} \sim \langle\Delta Z\rangle^{\zeta^\star(\lambda)\zeta^\star(p)}. \tag{8}$$

Then

$$\zeta^\star(\lambda p) = \zeta^\star(\lambda)\zeta^\star(p). \tag{9}$$

This functional equation has the general solution

$$\zeta^\star(p) = p^\gamma \tag{10}$$

where $\gamma$ is some constant. Then from (2), (4) and (10) we obtain

$$\zeta_p = \zeta_1 p^\gamma \tag{11}$$

and, consequently,

$$H_p \sim p^{(\gamma-1)}. \tag{12}$$

The solid straight line (best fit) is drawn in figure 2 to indicate dependence (12) with multifractal exponent $\gamma \simeq 0.556$ for the long-time data (filled circles).

Generally, it is a very difficult mathematical problem to reconstruct the probability distribution using the moment's properties. It seems from direct calculations of corresponding probability distributions that a power-law-like distribution with a stretched exponential tail can be a candidate for this role (cf [12]).

Finally, let us discuss briefly the obtained results. For comparatively short times (up to five months) the above-discussed differences between 'old' and 'new' economics have no significant (statistical) influence on the pricing process and the standard Black–Scholse pricing theory (based on the quasi-Brownian diffusion) can be applied in the short-time horizons. However, for comparatively long-time processes the influence of the new 'expectation' mechanism on the returns in NASDAQ becomes crucial. If for the 'old' indexes quasi-Brownian diffusion (with $\gamma = 1$) is usually replaced by multiplicative cascades in the multifractal horizons (that results in log-normal-like probability distributions with still *integer* $\gamma = 2 > 1$), for the NASDAQ index (with its non-integer $\gamma \simeq 0.556 < 1$) a significant non-analytic behaviour of the generalized Hurst exponents $H_p$ can be seen from (12). This means, in particular, that adequate pricing theory for the long-time NASDAQ index cannot be reduced to the existing pricing theories based on the analytic dependence of $H_p$ on $p$.

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